Time : 3 hours Max Marks : 60

DIFFERENTIAL GEOMETRY II - END-SEMESTRAL EXAM.

Answer all questions. You may use results proved in class after correctly quoting them. Any other claim must be accompanied by a proof.

(1) Show that $O(n)$, the set of orthogonal matrices is a sooth manifold, find its diension and the tangent space at $A \in O(n)$. [10]

(2) Define the Lie bracket of two vector fields. Given a smooth map $f : M \longrightarrow N$ between manifolds, two vector fields $X \in \mathfrak{X}(M)$ and $Y \in \mathfrak{X}(Y)$ are f-related if

$$df \circ X = Y \circ f.$$

Show that if $X_i \in \mathfrak{X}(M)$ is f-related to $Y_i \in \mathfrak{X}(Y)$, $i = 1, 2$, then $[X_1, X_2]$ is f-related to $[Y_1, Y_2]$. [2+8]

(3) Let X be the vector field on $\mathbb{R}^2$ defined by

$$X = x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}.$$

Find the maximal integral curve of X through $p = (a, b)$. Is X complete when thought of as a vector field on $\mathbb{R}^2 - 0$, on $\mathbb{R}^2 - (1, 0)$? [6+6]

(4) Discuss the definitions of : left invariant vector field on a Lie group, the exponential map of a Lie group. Describe in detail the exponential map of the Lie group S^1 of complex numbers of modulus 1. [4+8]

(5) Let g denote the Riemannian metric on S^2 induced from the usual metric on $\mathbb{R}^3$. Describe, with complete details, the Levi-Civita connection associated to the metric g and the curvature tensor. Compute the Gauss curvature. [8+4+4]